

Electron transmission in normal/heavy-fermion superconductor junctions

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The Andreev reflection between a normal metal (N) and a heavy-fermion superconductor (HFS) is studied and the boundary conditions for the electron's wave function in the two systems are established in the framework of a two band model for the HFS. Hence we show in a simple and explicit way that the mass enhancement factors in the heavy-fermion (HF) metal do not cause impedance at the N/HFS interface, in accordance with arguments previously presented. We also present an extension of the theory to a two-fluid model for the heavy-fermion, as possibly applicable to *e.g.*, CeCoIn₅.

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I. INTRODUCTION AND MODEL

Electronic scattering at the interface between a normal metal (N) and a superconductor has been studied by Blonder, Tinkham and Klapwijk¹ (BTK) and it was found that the subgap conductance is enhanced due to Andreev reflection. On the other hand, the Fermi velocity mismatch between the two metals always produces an effective barrier which decreases the conductance^{1,2}. If the superconductor is a heavy-fermion (HFS), the greater Fermi velocity mismatch would lead us to expect a strongly reduced subgap conductance. Experimentally, however, the subgap conductance does not seem to be strongly reduced in N/HFS junctions. An argument to explain this behavior has been put forward by Deutcher and Nozières³, who claimed that mass enhancement factors in the heavy-fermion metal do not cause impedance at the interface.

Motivated by recent experiments^{4,5} on Au/CeCoIn₅ interfaces, we study electron scattering at the interface between a normal (light) metal (N) and a heavy-fermion superconductor. Starting from a more realistic two-band model for the HFS, where a conduction *c*-electron band hybridizes with a localized *f*-electron band, plane-wave solutions for each bulk subsystem can be written down. We explicitly obtain the matching conditions for the wave function at the interface and confirm the claim in Ref³, by explicitly showing that the electron velocities involved are those of the conduction *c*-bands. Since the localized *f*-electrons are dispersionless, only the *c*-conduction electrons satisfy matching conditions at the interface. This is the basis of the result.

An extension to a two-fluid model of the HFS, where a normal light fluid coexists with the superconducting one, is also presented, which may be relevant to *e.g.*, CeCoIn₅. Our theory is analogous to the quantum waveguide theory for mesoscopic structures⁶.

The model for the HFS is:

$$\hat{H}_{hf} = \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} (\epsilon_f - \mu) f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma}$$

$$+ V \sum_{\mathbf{k}\sigma} \left(f_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + h.c. \right) + \sum_{\mathbf{k}} \left(\Delta_{\mathbf{k}} f_{\mathbf{k}\uparrow}^\dagger f_{-\mathbf{k},\downarrow}^\dagger + h.c. \right). \quad (1)$$

The superconducting pairing, $\Delta_{\mathbf{k}}$, has been explicitly written for the *f*-electrons because such a model qualitatively reproduces the generic phase diagram of a HFS^{7,8,9}. Equation (1) is an effective model obtained in the large Anderson-*U* limit, where it is understood that both the *f*-electron level, ϵ_f , and the hybridization, *V*, are obtained self-consistently.

The Bogolubov operators which diagonalize the above Hamiltonian are given by:

$$\hat{\gamma}_{\mathbf{k}\sigma} = u \hat{c}_{\mathbf{k}\sigma} - \sigma v^* \hat{c}_{-\mathbf{k}-\sigma}^\dagger + \tilde{u} \hat{f}_{\mathbf{k}\sigma} - \sigma \tilde{v}^* \hat{f}_{-\mathbf{k}-\sigma}^\dagger, \quad (2)$$

where the amplitudes obey the linear system

$$\mathbf{H}(\mathbf{k}) \begin{pmatrix} u \\ \tilde{u} \\ v \\ \tilde{v} \end{pmatrix} = E(\mathbf{k}) \begin{pmatrix} u \\ \tilde{u} \\ v \\ \tilde{v} \end{pmatrix}, \quad (3)$$

with

$$\mathbf{H}(\mathbf{k}) = \begin{pmatrix} \epsilon_{\mathbf{k}} - \mu & V & 0 & 0 \\ V & \epsilon_f - \mu & 0 & \Delta_{\mathbf{k}} \\ 0 & 0 & -\epsilon_{\mathbf{k}} + \mu & -V \\ 0 & \Delta_{\mathbf{k}}^* & -V & -\epsilon_f + \mu \end{pmatrix}. \quad (4)$$

The excitation energies are given by:

$$E(\mathbf{k}) = \sqrt{\gamma(\mathbf{k}) \pm \sqrt{\gamma^2(\mathbf{k}) - \beta^2(\mathbf{k})}}, \quad (5)$$

with

$$2\gamma(\mathbf{k}) = (\epsilon_f - \mu)^2 + (\epsilon_{\mathbf{k}} - \mu)^2 + |\Delta_{\mathbf{k}}|^2 + 2V^2, \quad (6)$$

$$\beta^2(\mathbf{k}) = (\epsilon_{\mathbf{k}} - \mu)^2 |\Delta_{\mathbf{k}}|^2 + [(\epsilon_f - \mu)(\epsilon_{\mathbf{k}} - \mu) - V^2]^2. \quad (7)$$

For convenience, we introduce the simplified notation:

$$\xi = \epsilon_{\mathbf{k}} - \mu, \quad \epsilon = \epsilon_f - \mu, \quad \Delta = \Delta_{\mathbf{k}}. \quad (8)$$

The linear system (3)-(4) yields a relation between the amplitudes on the f and c subsystems:

$$\tilde{u} = \frac{E - \xi}{V} u, \quad \tilde{v} = -\frac{E + \xi}{V} v. \quad (9)$$

From (9) we see that the amplitudes on the f sites are not independent being proportional to the amplitudes on the c sites. The description of the bands is standard and is briefly reviewed in the Appendix.

II. ELECTRON TRANSMISSION

An incident electron from the light metal, with energy $E \geq 0$ measured from the Fermi level, will penetrate the HFS. The coherence factors for the transmitted quasiparticles (quasi-hole and quasi-electron) can be obtained from the linear system (3)-(4). Using (9) to eliminate \tilde{u} , \tilde{v} we get:

$$\begin{aligned} [V^2 - (E - \xi)(E - \epsilon)] u - \Delta \cdot (E + \xi) v &= 0 \\ \Delta \cdot (\xi - E) u + [V^2 - (E + \xi)(E + \epsilon)] v &= 0 \end{aligned} \quad (10)$$

Equating the corresponding determinant to zero we obtain:

$$\xi = \frac{\epsilon V^2 \pm \sqrt{-\Delta^2 V^4 + E^2 (\epsilon^2 + \Delta^2 + V^2 - E^2)^2}}{\epsilon^2 + \Delta^2 - E^2} \quad (11)$$

Equation (11) determines the momenta of the transmitted quasiparticles (equation (19) below).

At the Fermi level ($E = 0$), the quasi-particles decay exponentially into the HFS if the argument of the square root is negative (roughly if $E < \Delta_k$, for a specific direction). Equation (11) then gives

$$\xi = \frac{\epsilon V^2 \pm \sqrt{-\Delta^2 V^4}}{\epsilon^2 + \Delta^2} \approx \frac{V^2}{\epsilon} \pm i \frac{V^2}{\epsilon^2} \Delta \approx \xi_F \pm i \hbar v_F^{(c)} \kappa, \quad (12)$$

where we have defined

$$\kappa = \frac{\Delta}{\hbar v_F^{(c)}} \left(\frac{V}{\epsilon} \right)^2. \quad (13)$$

It is seen from equation (13) that the momentum on the decaying quasiparticle in the HFS has a real part which is k_F and an imaginary part, κ . The decay length, κ^{-1} , is determined by the *slow* Fermi velocity of the heavy-fermion system, $v_F^{(c)} \epsilon^2 / V^2$.

We write the electron wave function in the N/HFS system as a four-component vector by generalizing the column vector in equation (3):

$$\Psi(\mathbf{r}) = \begin{pmatrix} f(\mathbf{r}) \\ \tilde{f}(\mathbf{r}) \\ g(\mathbf{r}) \\ \tilde{g}(\mathbf{r}) \end{pmatrix}.$$

Inside the normal (light) metal we have $\tilde{f} = \tilde{g} = 0$, of course. We identify the N/HFS interface with the $x = 0$ plane. When a plane wave is scattered at the interface, the wavevector component that is parallel to the interface is conserved. While no boundary conditions are imposed on \tilde{f} and \tilde{g} , the functions f and g are required to be continuous at the NS boundary. Allowing for two different c -band effective masses in the two metals, we write the kinetic energy operator as^{3,10}

$$\hat{T} = -\frac{\hbar^2}{2} \left[\partial_x \frac{1}{m(x)} \partial_x + \frac{1}{m(x)} (\partial_y^2 + \partial_z^2) \right], \quad (14)$$

where $m(x) = m_n$ in the light metal ($x < 0$) and $m(x) = m_{hf}$ in the heavy metal ($x > 0$). The mass m_{hf} is not large: it is simply the c -band effective mass. Because of the above form of the kinetic energy, the derivative of ψ is not continuous at $x = 0$. The functions f , \tilde{f} and g , \tilde{g} are coupled by the equations:

$$Ef = \left[\hat{T} - \mu + U\delta(x) \right] f + V\Theta(x)\tilde{f} \quad (15)$$

$$Eg = \left[-\hat{T} + \mu - U\delta(x) \right] g - V\Theta(x)\tilde{g}, \quad (16)$$

where U denotes a potential barrier at the interface and $\Theta(x)$ denotes Heaviside's function. The remaining boundary condition for f and g is obtained upon integration of (15)-(16) between $x = 0^-$ and $x = 0^+$:

$$\frac{\hbar^2}{2} \left[\frac{f'(x=0^+)}{m_{hf}} - \frac{f'(x=0^-)}{m_n} \right] = Uf(0) \quad (17)$$

$$\frac{\hbar^2}{2} \left[\frac{g'(x=0^+)}{m_{hf}} - \frac{g'(x=0^-)}{m_n} \right] = Ug(0). \quad (18)$$

where the prime denotes derivation with respect to x .

We write $\Psi(\mathbf{r}) = e^{i\mathbf{k}_{||} \cdot \mathbf{r}_{||}} \psi(x)$ where $\mathbf{k}_{||}$ and $\mathbf{r}_{||}$ are the wavevector and position vector components parallel to the interface, and

$$\begin{aligned} \psi(x < 0) &= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{ip^+x} + b \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ip^+x} + a \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ip^-x}, \\ \psi(x > 0) &= c \begin{pmatrix} u_+ \\ \tilde{u}_+ \\ v_+ \\ \tilde{v}_+ \end{pmatrix} e^{ik^+x} + d \begin{pmatrix} u_- \\ \tilde{u}_- \\ v_- \\ \tilde{v}_- \end{pmatrix} e^{-ik^-x}, \end{aligned} \quad (19)$$

where b denotes the particle-particle reflection amplitude and a denotes the Andreev reflection amplitude. The transmitted quasi-electron and quasi-hole in the HFS have amplitudes c and d , respectively. The momenta are obtained from energy conservation: $E(k_{||}, k^-) = E(k_{||}, k^+) = -\xi_n(k_{||}, p^-) = \xi_n(k_{||}, p^+) = E$, where $E(\mathbf{k})$ is given in equation (A5) and $\xi_n(\mathbf{k})$ denotes the incident electron dispersion in the normal metal measured from the Fermi level.

In the following we calculate the reflection amplitudes a and b for $E = 0$ and normal incidence ($p_{||} = 0$), for

simplicity. In this case all functions depend on the coordinate x , only. Equations (10)-(11) give $v_+ = -iu$ and $k^+ = k_F + i\kappa$ for the transmitted quasi-electron, and $v_- = iu$ and $k^- = k_F - i\kappa$ for the transmitted quasi-hole. Owing to the different c band effective masses, we denote the Fermi velocity in the light metal by v_n , which should be comparable to that obtained from the slope of the c band in the HF, $v_F^{(c)}$. The continuity of the functions f and g at the interface implies:

$$1 + b = cu + du, \quad (20)$$

$$ia = cu - du, \quad (21)$$

and equations (17)-(18) imply:

$$\frac{2U(1+b)}{\hbar^2} + \frac{ip_F(1-b)}{m_n} = cu \frac{ik^+}{m_{hf}} - du \frac{ik^-}{m_{hf}}, \quad (22)$$

$$\left(\frac{2U}{\hbar^2} + \frac{ip_F}{m_n} \right) a = cu \frac{k^+}{m_{hf}} + du \frac{k^-}{m_{hf}}, \quad (23)$$

where $\hbar p_F$ denotes the Fermi momentum in the normal metal, $v_n = \hbar p_F / m_n$ and $v_F^{(c)} = \hbar k_F / m_{hf}$. Introducing the dimensionless barrier parameter¹

$$Z = \frac{U}{\hbar v_n}, \quad (24)$$

we may write the reflection amplitudes as:

$$a = \frac{-2i \frac{v_F^{(c)}}{v_n}}{1 + \left(\frac{v_F^{(c)}}{v_n} \right)^2 + \left(2Z + \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2}, \quad (25)$$

$$b = \frac{\left(1 - 2iZ - i \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2 - \left(\frac{v_F^{(c)}}{v_n} \right)^2}{1 + \left(\frac{v_F^{(c)}}{v_n} \right)^2 + \left(2Z + \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2}. \quad (26)$$

The expressions (25) and (26) are precisely what would be obtained if the HFS was a one-band superconductor with a Fermi velocity $v_F^{(c)}$. This can be traced back to the fact that only the c -electron parts of the wave function, $f(\mathbf{r})$, and $g(\mathbf{r})$, satisfy matching conditions at the interface that are the same as in the case of a one-band superconductor. Inside the HFS, the f -site amplitudes $\tilde{f}(\mathbf{r})$ and $\tilde{g}(\mathbf{r})$ are directly proportional to the c -electron amplitudes as shown in (9). The mass enhancement, or slow Fermi velocity, in the HF appears in (25) and (26) through the coherence length κ , as can be seen from (13). In the case of a clean interface ($Z = 0$) and long decay length (such as close to a nodal direction), $\kappa \rightarrow 0$, these expressions simplify to

$$a = \frac{-2i\eta}{1 + \eta^2}, \quad b = \frac{1 - \eta^2}{1 + \eta^2}, \quad (27)$$

where $\eta = v_F^{(c)} / v_n$. If there is no mismatch of light Fermi velocities ($\eta = 1$) then $b = 0$ and $|a|^2 = 1$ leading in this rather special case to a perfect doubled conductance: $1 + |a|^2 - |b|^2 = 2$.

III. TWO-FLUID HFS

A theory for electron transmission from a normal metal to a one-band superconductor¹¹ was applied to an interface with the HFS CeCoIn₅^{4,5}. This material has a complex Fermi surface¹² and seems to be well described by a two-fluid model^{5,13,14}. The superconducting gap has d -wave symmetry, as determined directly by various experiments^{5,15,16,17}.

The HF metal CeCoIn₅ is known to have at least one band of uncondensed light carriers in addition to the heavy fermion superconducting liquid. Recent Andreev reflection studies assume that the subgap conductance is a weighted average between that given by the BTK theory and a flat conductance due to the band of uncondensed fermions⁵. This motivates us to analyze the case where the incident electrons from the normal metal (labeled as system "1") can tunnel simultaneously to a c -band coupled to the f subsystem (labeled "2") and to a light uncondensed conduction band (labeled "3"), co-existing in the heavy-fermion material. Metal 1 is in the $x < 0$ half-space and metals 2 and 3 are in the $x > 0$ half-space, with the interface at $x = 0$. The theory we employ is an extension of the quantum waveguide theory of mesoscopic structures⁶ where a delta function potential is introduced at the interface and where one of the circuit branches, "2", is the HFS model of equation (1).

The potential $U\delta(x - \varepsilon)$ is in metal 1 ($x < 0$) and we shall take the limit $\varepsilon \rightarrow 0^-$. The wave function in the normal single-band metal 1 has particle and hole components¹⁸:

$$\psi_1(x \leq \varepsilon) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ip^+x} + b \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ip^+x} + a \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ip^-x}, \quad (28)$$

and

$$\begin{aligned} \psi_1(x \geq \varepsilon) = & \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{ip^+x} + \beta \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-ip^+x} \\ & + \gamma \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{ip^-x} + \delta \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-ip^-x}. \end{aligned} \quad (29)$$

The wave function in metal 2 is written in the same form as (19):

$$\psi_2(x \geq 0) = c \begin{pmatrix} u_+ \\ \tilde{u}_+ \\ v_+ \\ \tilde{v}_+ \end{pmatrix} e^{ik^+x} + d \begin{pmatrix} u_- \\ \tilde{u}_- \\ v_- \\ \tilde{v}_- \end{pmatrix} e^{-ik^-x} \quad (30)$$

and for the normal metal 3 we simply write a transmitted electron and hole:

$$\psi_3(x \geq 0) = t \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq^+x} + t_a \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iq^-x}. \quad (31)$$

The matching conditions at $x = \varepsilon < 0$ give the equations:

$$\psi_1(\varepsilon^-) = \psi_1(\varepsilon^+), \quad (32)$$

$$-\frac{\hbar^2}{2m_n} [\psi_1'(\varepsilon^+) - \psi_1'(\varepsilon^-)] + U\psi_1(\varepsilon) = 0. \quad (33)$$

Taking the limit $\varepsilon \rightarrow 0^-$ we obtain:

$$\alpha + \beta = 1 + b, \quad (34)$$

$$\alpha - \beta = \frac{2m_n U}{i\hbar^2 p^+} (1 + b) + 1 - b, \quad (35)$$

$$\gamma + \delta = a, \quad (36)$$

$$\gamma - \delta = \left(\frac{2m_n U}{i\hbar^2 p^-} + 1 \right) a. \quad (37)$$

We now proceed with the matching condition at $x = 0$ using the theory in Ref⁶. The single-valuedness of the wave function at $x = 0$ implies that:

$$\alpha + \beta = cu_+ + du_-, \quad (38)$$

$$\alpha + \beta = t, \quad (39)$$

$$\gamma + \delta = cv_+ + dv_-, \quad (40)$$

$$\gamma + \delta = t_a, \quad (41)$$

and the (probability) current conservation implies that:

$$\frac{ip^+}{m_n} (\alpha - \beta) = \frac{ik^+}{m_{hf}} cu_+ - \frac{ik^-}{m_{hf}} du_- + \frac{iq^+}{m} t, \quad (42)$$

$$\frac{ip^-}{m_n} (\gamma - \delta) = -\frac{ik^+}{m_{hf}} cv_+ - \frac{ik^-}{m_{hf}} dv_- - \frac{iq^-}{m} t_a, \quad (43)$$

where m denotes the electron's effective mass in metal 3. Equations (34)-(37) allow the elimination of the amplitudes $\alpha, \beta, \gamma, \delta$.

These equations can be applied again to the case of normal incidence at $E = 0$. Defining the Fermi momentum and velocity in metal 3 as $\hbar q_F$ and $v_3 = \hbar q_F/m_3$, respectively, we obtain modified results for the amplitudes a and b :

$$a = \frac{-2i \frac{v_F^{(c)}}{v_n}}{1 + \left(\frac{v_F^{(c)}}{v_n} \right)^2 + \left(2Z + \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2 + \frac{v_3}{v_n} \left(\frac{v_3}{v_n} + 2 \right)} \quad (44)$$

$$b = \frac{\left(1 - 2iZ - i \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2 - \left(\frac{v_F^{(c)}}{v_n} \right)^2 - \left(\frac{v_3}{v_n} \right)^2}{1 + \left(\frac{v_F^{(c)}}{v_n} \right)^2 + \left(2Z + \frac{v_F^{(c)}}{v_n} \frac{\kappa}{k_F} \right)^2 + \frac{v_3}{v_n} \left(\frac{v_3}{v_n} + 2 \right)} \quad (45)$$

In the limiting case of a clean junction with the normal incidence close to the nodal direction, $Z, \kappa \rightarrow 0$, the above expressions simplify to

$$a = \frac{-2i\eta}{1 + \eta^2 + \eta'(\eta' + 2)}, \quad (46)$$

$$b = \frac{1 - \eta^2 - \eta'^2}{1 + \eta^2 + \eta'(\eta' + 2)},$$

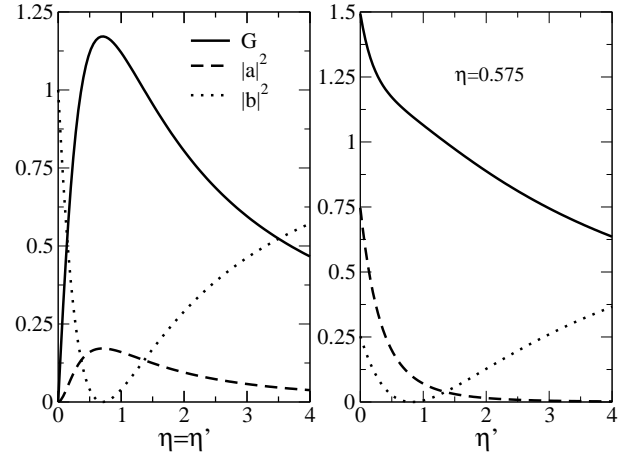


FIG. 1: Scattering amplitudes (46) and conductance $G = 1 + |a|^2 - |b|^2$ for $Z, \kappa = 0$ as functions of η' for: $\eta = \eta'$ (left panel); $\eta = 0.575$ (right panel), which is the value inferred from the effective barrier parameter estimated in Ref.⁵.

where $\eta = v_F^{(c)}/v_n$, as in equation (27), and $\eta' = v_3/v_n$. In order to obtain the perfect conductance for this case, we assume no mismatch of light Fermi velocities ($\eta = \eta' = 1$) so that $|b|^2 = 0.04$ and $|a|^2 = 0.16$. The perfect conductance in this case is then

$$G_{\text{perfect}} = 1 + |a|^2 - |b|^2 = 1.12, \quad (47)$$

which should be compared to the well known result $1 + |a|^2 - |b|^2 = 2$ in BTK theory. The effect of metal 3 is to "short-circuit" the system by providing a new transmission channel for the incoming electron. In this case the electron does not need to pick up a second electron and penetrate the superconductor as a Cooper pair, leaving the Andreev hole behind. Hence the suppression of the Andreev reflection process from $|a|^2 = 1$ to $|a|^2 = 4/25$. The effect of metal 3 is reduced if the ratio v_3/v_n decreases and the theory of section II is recovered in the limit $v_3/v_n \rightarrow 0$. In the opposite limit, $v_3/v_n \rightarrow \infty$ implies $a \rightarrow 0$ and $b \rightarrow -1$. Figure 1 shows a plot of equations (46) and the conductance for some choices of velocity ratios.

We note that the result (47) is remarkably close to the enhanced subgap conductance observed in CeCoIn₅, where the zero-bias differential conductance^{4,5} lies in the range 1-1.13. Figure 1 shows that some choices of velocity ratios yield conductances in this range. A fit of our model to the extensive experimental data on this compound might settle the model parameters values.

We also note that the form of equations (44)-(45) shows that the conductance cannot be written a sum of two independent parallel conductances, one from the heavy liquid, another from the light one. The interface impedance here is a quantum-mechanical effect due to the boundary conditions for the wave function at an intersection, as in quantum waveguide theory.

IV. CONCLUSIONS

The success of the two-fluid model used in Ref.⁵ to explain the conductance spectra of the junction Au/CeCoIn₅ relies on the assumptions that: (i) only the light velocities are important; (ii) the density of states is decreasing as one approaches the Fermi level; (iii) the conductance may be obtained as a weighted average. Here we have explicitly confirmed the validity of (i). Item (ii) is not consistent with the usual approach where one considers the lowest band partially filled, but is consistent if we consider that the lowest band is full and the Fermi level is located at the bottom of the higher band. In this regime the effective mass is also high and the density of states is decreasing with energy. However, only the light velocities affect the Andreev reflection. Regarding (iii), we have shown that in general the conductance may not be written as a sum since the interface impedance is a quantum-mechanical effect which leads to interference terms.

V. ACKNOWLEDGMENTS

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APPENDIX A: ANALYSIS OF EQ. (5)

If $\Delta = 0$ and the system is less than half-full, all electrons are in the lower band given by

$$E_-(\mathbf{k}) = \frac{1}{2} \left[\xi + \epsilon - \sqrt{(\xi - \epsilon)^2 + 4V^2} \right]$$

The Fermi level is given by $E_- = 0$, which means that $\beta=0$ in equations (5) and (7), or:

$$\xi_F = \frac{V^2}{\epsilon} \quad (\text{A1})$$

The Fermi velocity is then given by

$$\frac{dE_-}{dk} = \left(\frac{dE_-}{d\xi} \right)_F \left(\frac{d\xi}{dk} \right)_F = \frac{1}{1 + \left(\frac{V}{\epsilon} \right)^2} \hbar v_F^{(c)} \quad (\text{A2})$$

where $v_F^{(c)} = \hbar^{-1} (d\xi/dk)_F$ denotes the c -band velocity evaluated at the Fermi momentum $\hbar k_F$, and is comparable to that of the light metal. In order for the density of states (or mass enhancement) to be high, we must have $V^2 \gg \epsilon^2$. Figure 2 shows the bands in the normal state of the HF and the relevant energy scales.

For finite Δ , the lowest band of excitation energies is obtained by choosing the minus sign in equation (5), which may be written as

$$E_-^2 = \gamma_0 + \frac{\Delta^2}{2} - \sqrt{\left(\gamma_0 + \frac{\Delta^2}{2} \right)^2 - (\beta_0^2 + \Delta^2 \xi^2)} \quad (\text{A3})$$

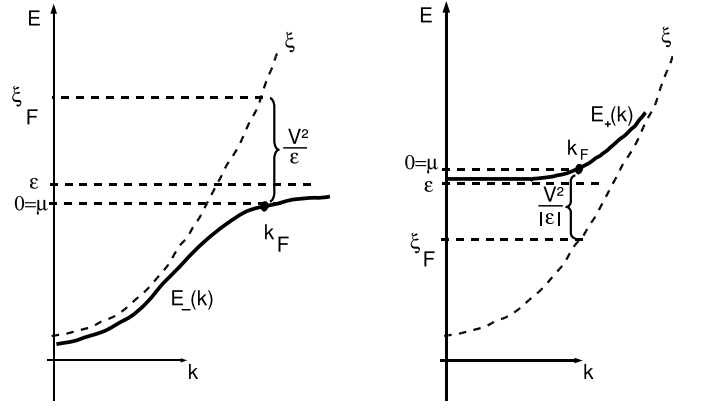


FIG. 2: Left: the lower partially occupied band $E_-(\mathbf{k})$ of a normal HF system resulting from the hybridization between the f and c subsystems with energies $\epsilon \equiv \epsilon_f - \mu$ and $\xi \equiv \epsilon_c - \mu$, respectively. The c band energy evaluated at the Fermi momentum, k_F , is ξ_F and has a slope $v_F^{(c)} = (d\xi/dk)_F$ which is larger than the actual Fermi velocity $\hbar^{-1} (dE_-/dk)_F = v_F^{(c)} \epsilon^2 / V^2$. Right: the upper partially occupied band $E_+(\mathbf{k})$.

where γ_0 and β_0 are the functions γ and β given in (6)-(7) when $\Delta = 0$. By expanding in Δ (assuming $V^2 \gg \epsilon^2 \gg \Delta^2$ which is valid for instance for CeCoIn₅ where the coherence temperature is of the order of 45 K and the superconducting critical temperature is of the order of 2.3 K) we obtain the gap for excitations at k_F :

$$E(\mathbf{k}_F) = \Delta(\mathbf{k}_F) \frac{(V/\epsilon)^2}{1 + (V/\epsilon)^2} \approx \Delta(\mathbf{k}_F) \quad (\text{A4})$$

and the excitation spectrum in the vicinity of the Fermi level is:

$$E(\mathbf{k}) = \sqrt{\left[\frac{\hbar v_F^{(c)} (|\mathbf{k}| - k_F)}{1 + (V/\epsilon)^2} \right]^2 + \Delta^2(\mathbf{k})}, \quad (\text{A5})$$

which has the usual form and involves the slow velocity of the heavy electrons.

We may as well consider a situation where the local Coulomb repulsion between the electrons is not too large, which enables a f -site occupancy $n_f > 1$. This case is also represented in Fig. 2, where we consider that the Fermi level is located above ϵ_f . In this case $\xi_F, \epsilon < 0$. The position of the Fermi level is again obtained from $\beta_0 = 0$ and leads in this case to $\xi_F = -V^2/|\epsilon|$. It is easy to see that the Fermi velocity is once again of the form (A2), as expected of a heavy band. Note however, that while for the case when $n_f < 1$ the density of states is increasing as we approach the Fermi level, in the case when $n_f > 1$ the density of states is a decreasing function of the energy in the vicinity of the Fermi level.

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